Excitation of Orbital Eccentricities of Extrasolar Planets by Repeated Resonance Crossings

E. I. Chiang, D. Fischer, & E. Thommes

Center for Integrative Planetary Sciences
Astronomy Department
University of California at Berkeley
Berkeley, CA 94720, USA

echiang, fischer, ethommes@astron.berkeley.edu

ABSTRACT

Orbits of known extrasolar planets that are located outside the tidal circularization regions of their parent stars are often substantially eccentric. By contrast, planetary orbits in our Solar System are approximately circular, reflecting planet formation within a nearly axisymmetric, circumsolar disk. We propose that orbital eccentricities may be generated by divergent orbital migration of two planets in a viscously accreting circumstellar disk. The migration is divergent in the sense that the ratio of the orbital period of the outer planet to that of the inner planet grows. As the period ratio diverges, the planets traverse, but are not captured into, a series of mean-motion resonances that amplify their orbital eccentricities in rough inverse proportion to their masses. Strong viscosity gradients in protoplanetary disks offer a way to reconcile the circular orbits of Solar System gas giants with the eccentric orbits of currently known extrasolar planets.

Subject headings: celestial mechanics — planetary systems — accretion, accretion disks

1. INTRODUCTION

Orbital eccentricities e, periods P, and semi-major axes a of extrasolar planetary systems are plotted in Figure 1. The rightmost four points at large P represent the gas and ice giants in our Solar System (Lodders & Fegler 1998). The leftmost cluster of points at small P reflect tidal interactions between planets and stars that erased whatever primordial eccentricities these systems possessed (Lin et al. 2000). In the intermediate range of periods, orbital

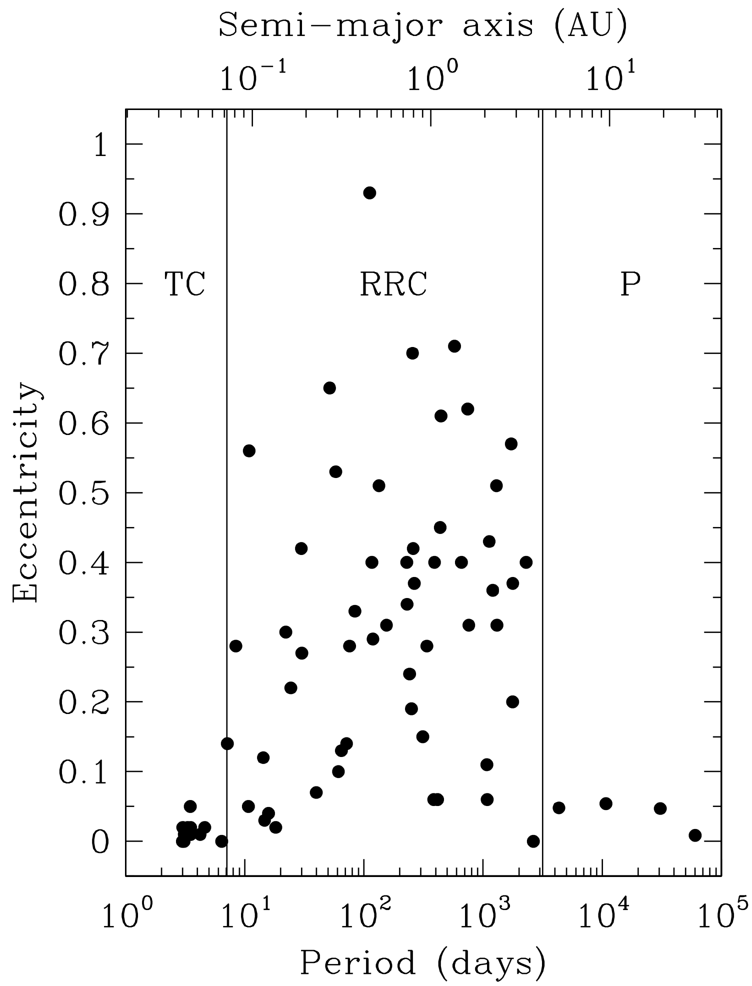

Fig. 1.— Orbital eccentricities and periods of 64 planets. The semi-major axis of the orbit is computed from the period using a central stellar mass of 1 $M_{\odot}$. TC represents the tidal circularization region, RRC the regime proposed to have witnessed repeated resonance crossings, and P the proposed outer passive region in which little or no migration occurred.

eccentricities can be strikingly large, typically exceeding those of giant planets in the Solar System by factors of ∼2–20.

Several theories have been proposed to explain these large eccentricities. Many encounter difficulties when applied to the majority of systems, and one remains incompletely developed. A stellar binary companion can secularly drive a planet's eccentricity (Holman, Touma, & Tremaine 1997), but nearly all known extrasolar planets orbit solitary stars. Alternatively, dynamical instabilities afflicting two planets formed at close separation on circular orbits can eject one planet while inducing a large orbital eccentricity in the remaining body (Rasio & Ford 1996; Weidenschilling & Marzari 1996). However, close encounters engineered in this fashion result also in planetary collisions, leaving a large proportion of planets on circular orbits that are not observed (Ford, Havlickova, & Rasio 2001). In a third scenario, orbital migration of two planets during which the orbital period of one planet approaches that of the other ("convergent migration") can lead to resonant capture and eccentricity pumping. While convergent migration and resonance capture likely underpin the orbital eccentricities of GJ 876b and GJ 876c, two planets observed to occupy a 2:1 resonance (Marcy et al. 2001b; Lee & Peale 2001), all other extrasolar planetary systems presently evince no mean-motion resonant behavior. A fourth explanation invokes gravitational interactions between planets and the disks from which they formed (e.g., Artymowicz 1998). The present theory of satellite-disk interactions has only been derived to lowest order in e (Goldreich & Tremaine 1980); it is inadequate when applied to extrasolar systems for which e's can be as large as 0.25–0.95. In the present theory, whether the disk damps or excites e depends sensitively on the distribution of disk material near the planet (Goldreich & Tremaine 1980; Papaloizou, Nelson, & Masset 2001). While this distribution is currently uncertain, Lee & Peale (2001) find that the disk must strongly damp eccentricities to reproduce the orbital parameters of GJ 876 and to avoid the fine-tuning problem of having the epoch of resonant capture coincide with the dissipation of the disk.

This Letter proposes a fifth mechanism for exciting orbital eccentricities: repeated resonance crossings of two planets migrating on divergent trajectories. In §2 we argue that divergent orbital migration of two planets is more likely than convergent migration. In §3 we demonstrate how divergent migration may lead to substantial eccentricity excitation. In §4 we highlight the requirements and qualitative predictions of our theory and areas for future work.

2. Disk-Driven Divergent Drift

Migration of planets can be driven by tidal interactions with their natal gaseous disks. Known extrasolar planets have sufficiently large masses ($M \gtrsim M_J$, where M_J is the mass of Jupiter) that they clear annular gaps in disk material about their orbits (Ward 1997). A planet that opens a gap is thereafter slaved to the viscous evolution of its host disk, and undergoes so-called "Type II" drift (Ward 1997). The disk and its embedded planet at stellocentric distance r slide towards the star on the viscous diffusion timescale,

$$t_D = r/|\dot{r}| \sim r^2/\nu \sim r^2/\alpha c_s h \sim 2 \times 10^4 \left(\frac{r}{1\,\mathrm{AU}}\right)^{1/2} \left(\frac{1000\,\mathrm{K}}{T}\right) \left(\frac{10^{-3}}{\alpha}\right) \mathrm{yr}\,. \tag{1}$$

Here $\nu = \alpha c_s h$ is the viscosity of the disk, c_s, h, and T are the sound speed, vertical scale height, and temperature of disk gas, respectively, and $0 \leq \alpha \leq 1$ measures the strength of angular momentum transport intrinsic to the disk. Equation (1) assumes a central stellar mass $M_* = M_\odot$.

The diffusion time $t_D \propto \sqrt{r}/T\alpha$ almost certainly increases with increasing r. Disk temperatures fall radially outwards as the stellar flux and the disk's absolute gravitational potential energy per unit mass diminish. Sources of angular momentum transport include (1) dissipation of density waves excited by numerous, densely nested planets that are insufficiently massive ($M \lesssim M_\oplus$, where $M_\oplus$ is the mass of the Earth) to open gaps (Goodman & Rafikov 2001), and (2) the magnetorotational instability (MRI) that afflicts sufficiently ionized disks (e.g., Stone et al. 2000). Mechanism 1 is capable of generating $\alpha \lesssim 10^{-3}$, where the exact value depends on the spatial density of small, as yet undetectable planets. Mechanism 2 has been demonstrated to generate $\alpha \sim 10^{-5}$–10^{-1}, the exact value correlating positively with the electrical conductivity of disk gas (Fleming, Stone, & Hawley 2000). The conductivity decreases with decreasing temperature, so that under mechanism 2, $d\alpha/dr < 0$.

The standard MRI operates only in disk regions $r \lesssim r_d$ that are sufficiently hot, $T \gtrsim 1000\,\mathrm{K}$, that thermal ionization of trace metals and sublimation of dust grains permit the magnetic Reynolds number to exceed the threshold required for instability. Accretion disk models place r_d between ∼0.1 AU and ∼1 AU (Gammie 1996; Bell et al. 1997; D'Alessio et al. 1998)—distances at which many extrasolar planets are presently located (see Figure 1). In the absence of mechanism 1, it is possible that α is effectively zero at $r \gtrsim 1\,\mathrm{AU}$. While Gammie (1996) has proposed that the standard MRI can still operate wherever gas densities are sufficiently low that Galactic cosmic rays can provide the requisite ionization levels, the likelihood of this prospect remains unclear for two reasons: (1) dust grains can severely reduce the electron density, and (2) even neglecting dust, and even if the magnetic

Reynolds number exceeds the critical value required for instability, the time required for a neutral molecule to collide with an ion is typically longer than a dynamical time, so that the bulk of the mostly neutral gas fails to accrete with the ions (Blaes & Balbus 1994). We return to the possibility of a static outer disk at the end of this Letter.

What is important for what follows are the two recognitions that gap-opening planets at $r \lesssim$ a few AU drift inwards at rates that are (1) extremely slow compared to local orbital frequencies, and (2) different. Since $dt_D/dr > 0$, two gap-opening planets at $r \lesssim$ a few AU drift inwards such that the ratio of the period of the outer planet, P_2, to that of the inner planet, P_1, grows. The divergence of P_2/P_1 implies that a series of mean-motion resonances will be crossed. During each resonance crossing, the orbital periods of the two bodies are momentarily commensurable; that is, the ratio of their orbital periods approaches and then exceeds a ratio of small, positive integers.

3. Resonance Crossings on Divergent Orbits

By contrast with the case where the period ratio P_2/P_1 converges towards unity, the probability of resonant capture is zero for diverging orbits (Sinclair 1972; Henrard & Lemaitre 1983; Peale 1986; Yu & Tremaine 2001; and references therein). Nonetheless, as in the convergent case, each divergent passage can substantially alter the orbital eccentricities (Henrard & Lemaitre 1983; Peale 1986; Malhotra 1988; Dermott, Malhotra, & Murray 1988) and semi-major axes of the migrating bodies.

We illustrate the underlying mechanics by considering the problem of a massive, inwardly migrating planet about a star, and its effect on a massless test particle on a more distant, co-planar orbit. We take the mass of the planet M_1 to equal $1.5 \times 10^{-3} M_*$, where $M_* = M_\odot$ is the mass of the central star. Referenced to a coordinate system fixed on the star, the initial orbital semi-major axis and eccentricity of the planet are $a_1 = 1\,\mathrm{AU}$ and $e_1 = 0$, respectively. In addition to feeling the Newtonian force of gravity exerted by the star, the planet feels an additional drag force $\vec{F}_{drag} = -M_1\vec{v}/t_{drag}$, where $\vec{v}$ is the instantaneous velocity of the planet and $t_{drag} = 1.6 \times 10^3\,\mathrm{yr}$ is the timescale over which a_1 decays to 0. This prescribed drag force is introduced to simulate the effects of disk-induced migration and does not directly affect the planet's eccentricity (Papaloizou & Larwood 2000). Our value for t_{drag} is chosen to illuminate the evolution on timescales that are not too long compared to P_2; larger values of t_{drag} are probably more realistic and will be considered later. The test particle is initially placed on an orbit having semi-major axis $a_2 = 1.35$ AU and eccentricity $e_2 = 0$, and is initially positioned at an angle $\Delta = 180°$ away from the angular position of the planet. The test particle feels only the gravitational attraction from the

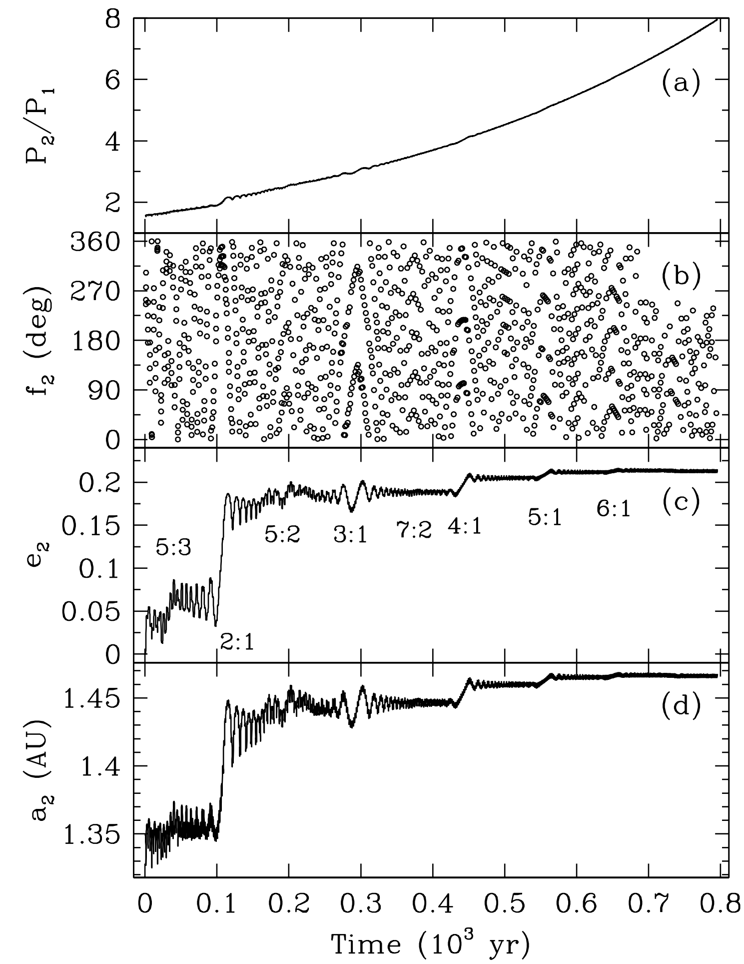

Fig. 2.— Anatomy of resonance passages involving an inwardly migrating massive planet and a massless test particle on a more distant, co-planar orbit. In panel (a) the ratio of orbital periods of the test particle (P_2) and of the planet (P_1) is plotted against time. Panel (b) plots the true anomaly of the test particle whenever the particle comes within 20° of the planet. Panels (c) and (d) plot the evolution of the test particle's eccentricity and semi-major axis. The resonances responsible for abrupt changes in the eccentricity are labelled.

planet and from the star. The subsequent positions and velocities of the planet and of the test particle are calculated using a variable-order, variable-step Adams numerical integration scheme (Hall & Watt 1976). Figure 2 displays the evolution. As the inner massive planet migrates towards the star, the mean eccentricity of the test particle undergoes 4 distinct changes. These changes occur when the period ratio P_2/P_1 equals 5/3, 2/1, 4/1, and 5/1. Concomitant changes in a_2 occur at these epochs of resonance passage. At times $t > 600\,\mathrm{yr}$, interactions between the particle and the now-distant planet are negligibly small, and the particle is left on a more eccentric ($e_2 = 0.21$) and slightly expanded ($a_2 = 1.47$ AU) orbit compared to its initial one.

These changes in e_2 and a_2 occur because during passage through a resonance, impulses of velocity are imparted to the test particle from the planet at specific phases in the test particle's orbit for extended periods of time. Accelerations felt by the test particle from the inner massive planet are strongest near times of conjunction, when positions of the star, planet, and test particle fall on a straight line and in that order. A $p : q$ resonance for which the planet executes integer p circular orbits for every integer q elliptical orbits traced by the test particle is characterized by $|p - q|$ conjunctions which occur at $|p - q|$ values of the particle's true anomaly f_2.[1] True anomalies f_2 near every conjunction are plotted in Figure 2b. For example, during passage through the 2:1 resonance, conjunctions are repeatedly occurring at values of f_2 concentrated in the interval between 270° and 360°. Conjunctions in this quadrant amplify e_2 and a_2 (Murray & Dermott 1999). Analytic expressions for the magnitudes of eccentricity jumps through first-order and second-order resonances are provided by Dermott, Malhotra, & Murray (1988).

Accounting for the finite mass of the outer body does not change our conclusions qualitatively. In Figure 3 we showcase 2 scenarios involving an inner body of mass $M_1/M_* = 2\times10^{-3}$ and an outer body of mass $M_2/M_* = 1 \times 10^{-3}$. To demonstrate that the substantial eccentricities that are excited in our simulation are caused by passages through resonances and not by the mere close proximity of these massive bodies, we do not impose any differential migration for the first $4 \times 10^3\,\mathrm{yr}$ of the integration. The osculating eccentricities of both bodies do not exceed 0.075 during this phase when δr, the instantaneous distance between the two planets, can be as small as 0.400 AU. Only when the drag force is applied to the inner planet at $t > 4 \times 10^3\,\mathrm{yr}$, using $t_{drag} = 1 \times 10^4\,\mathrm{yr}$, do the orbits diverge. The eccentricities of outer and inner bodies then undergo resonant excitation to values of $\sim$0.5 and $\sim$0.2, respectively, in the step-wise fashion characteristic of resonance crossings. The degree of amplification in e_2 exhibited in Figure 3 is greater than that in Figure 2 primarily because

[1]True anomaly measures the angular position of an object with respect to its periastron, and increases in the direction of the object's motion.

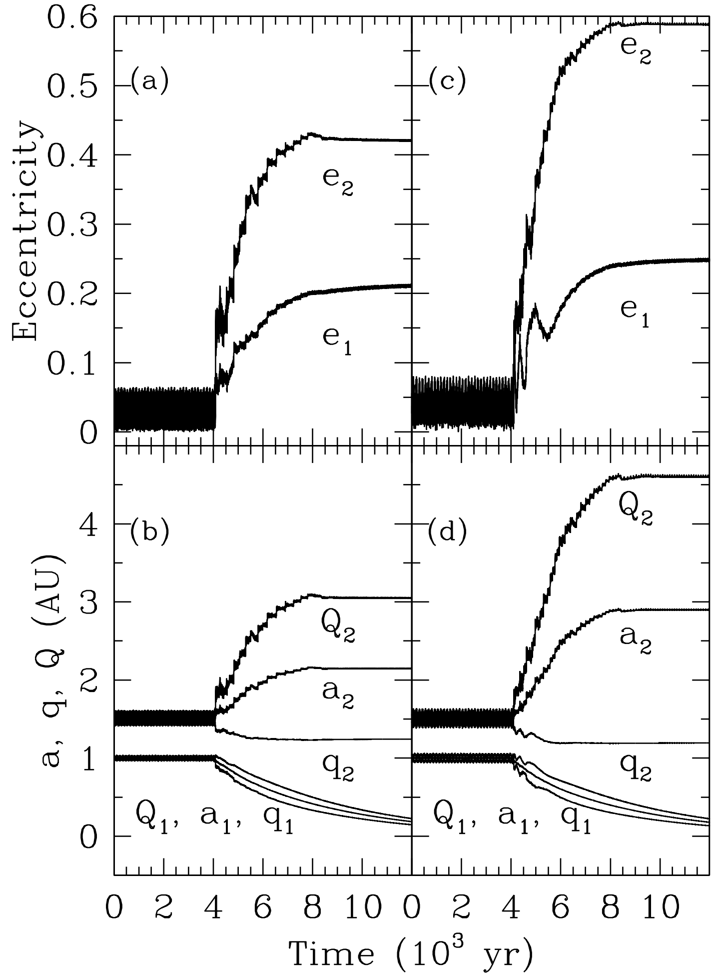

Fig. 3.— Resonance passages involving two massive planets. Masses of the inner and outer planets are $M_1/M_* = 2 \times 10^{-3}$ and $M_2/M_* = 1 \times 10^{-3}$, respectively. The drag force is applied to the inner planet over $t_{drag} = 1 \times 10^4$ yr starting at $t_{start} = 4 \times 10^3$ yr. At $t = 0$, the ratio of semi-major axes is $a_2/a_1 = 1.5$, the osculating eccentricities are both 0.005 (ab) or 0.03 (cd), and the planets are separated by an angle $\Delta = 180°$. At $t < t_{start}$, the planets mutually excite eccentricities of less than 0.075. Only after $t > t_{start}$ is differential migration introduced; the eccentricities of both objects become substantially excited through repeated resonance crossings. Panels (bd) also plot apastron distances $Q = a(1 + e)$ and periastron distances $q = a(1 - e)$.

the former is based on a longer t_{drag}. We have experimented with 20 different sets of initial orbital elements and find that eccentricities $e > 0.2$ are excited in all our test cases provided the initial $P_2/P_1 < 2$ so that the 2:1 resonance is crossed.

The eccentricity and semi-major axis jumps are due largely to resonant interaction and not to close encounters. The separation δr attains minimum values of 0.325–0.400 AU (Figures 3ab) and 0.306–0.400 AU (Figures 3cd) at $t = 4000$–$5000\,\mathrm{yr}$, whereas much of the amplification in e_2 and a_2 occurs over $t \approx 5000$–$6000\,\mathrm{yr}$ when $\delta r > 0.400\,\mathrm{AU}$. We have verified by a separate integration that freezing the divergent migration at $t = 4356\,\mathrm{yr}$ in the case of Figures 3cd—thereby freezing δr at its minimum value of 0.306 AU—does not generate further eccentricity or semi-major axis jumps at subsequent times, thus ruling out the significance of close encounters.

4. Discussion

We have established that divergent orbital migration of two planets can lead to significant eccentricity excitation without resonance capture. The effectiveness of this mechanism hinges on the detailed, and here umodelled, interaction between disk gas and the embedded planets. It is possible that the disk gas exerts torques on the planets on short enough timescales that the smooth navigation of the resonant separatrix is disrupted; conjunctions between the two planets may not occur at the same orbital phases for extended periods of time due to gravitational "noise" generated by disk gas. This is a concern that we defer to future hydrodynamic simulations of planet-disk interactions.

While we have staged our scenario within a gaseous circumstellar disk, it seems possible that divergent migration may also be effected by planetesimal scatterings (e.g., Murray et al. 1998; Hahn & Malhotra 1999). In their simulations of the formation of the Oort Cloud and the Kuiper Belt, Hahn & Malhotra (1999) find that Uranus and Neptune divergently migrate and cross a 2:1 resonance (see their Figure 6), thereby undergoing eccentricity excitation. The excitation is limited, however, because the planetesimal masses which they employ are large enough that the divergence of the planetary orbits is not adiabatic.

Our proposed mechanism operates most effectively when the initial orbits of the two bodies are sufficiently close that powerful resonances for which $|p - q| = 1$ (e.g., the 2:1 resonance) are crossed during subsequent migration. Formation of planets at such proximity is not unreasonable; a single giant planet embedded within a circumstellar disk may induce the collapse of a second planet in the vicinity of the 2:1 resonance (Armitage & Hansen 1999; Bryden et al. 2000).

Disk-driven divergent migration of two gap-opening planets is most effective if a ring of viscous disk material remains present between the two bodies. Kley (2000) and Bryden et al. (2000) have performed pioneering numerical simulations of two planets embedded in a disk resembling the minimum-mass solar nebula. While these calculations have tentatively shown that a ring can fail to be shepherded between two planets, so that planetary orbits converge rather than diverge, the outcome is model-dependent. The results of Bryden et al. (2000) suggest that if the mass of the ring is larger than the masses of the planets, or if the ring's intrinsic α is large so that planet-driven waves are efficiently dissipated near ring edges, then the ring can be confined; see, in particular, their model G.

The resonance crossings mechanism for generating eccentricities demands that each planet have at least one other planetary companion, either in the past or today. If $M_1 < M_2$, then $e_1 > e_2$, and vice versa. While the extrasolar planetary system υ And violates this requirement (Butler et al. 1999; Chiang, Tabachnik, & Tremaine 2001), the system HD 168433 (Marcy et al. 2001a) satisfies it. The absence of a second companion today in a given planetary system may indicate that the companion was accreted onto the star, either because its eccentricity grew too large by resonance crossings, or because "Type II" interactions drove the planet to its fiery destruction. A signature of planetary accretion would be an enhanced host star metallicity, but enhancements due to accretion of Jupiter-like giants during the first $\sim 10^7$ yr of the life of the star are too small to detect reliably (Murray et al. 2001).

The condition that the planets initially share the same orbit plane may be relaxed. We would expect a non-zero initial mutual inclination, i, to be amplified in analogous manner to the way eccentricities are excited. Lifting the planet out of the plane of the disk may represent a means of survival against continued migration. However, the degree of amplification in i is expected to be less than that in e because inclination resonances are at least second-order ($|p-q| \geq 2$) in strength (Murray & Dermott 1999). Calculating the relative orbital inclinations offers a means of testing our theory; future astrometric missions such as the Full-sky Astrometric Mapping Explorer (FAME) and the Space Interferometry Mission (SIM), in conjunction with stellar spectroscopic measurements, can place bounds on the degree of misalignment between orbital axes of eccentric planets and the spin axes of their parent stars.

If the MRI is the sole source of viscosity late in the life of a protoplanetary disk, we would expect gap-opening, Jupiter-mass planets at distances outside a few AU to have suffered little to no migration within the primordial gas disk. While it is unconventional to think of T Tauri disks as having $\alpha = 0$ at $r \gtrsim r_d \sim 1\,\mathrm{AU}$, such a model does not appear to violate observation or theory. It would require the disk to contain $\sim 0.01\ M_\odot$ inside $r \lesssim 1\,\mathrm{AU}$, a condition for which the disk remains gravitationally stable. Timescales for accretion of

this much material could be as long as those observed, $\sim 10^6$ yr, if $\alpha \sim 10^{-5}$ at $r \sim 1$ AU. Given a static outer disk, the theory of eccentricity excitation proposed here would predict that orbits of giant planets at $r \gtrsim$ a few AU be nearly circular. Giant planet orbits in our Solar System conform to this expectation. We await the results of ongoing Doppler velocity searches and future space-based interferometric surveys for extrasolar planets to confirm whether the orbital architecture of the outer Solar System is indeed commonplace.

We thank C. McKee for thoughtful comments on the manuscript and emboldening discussions, and P. Goldreich, R. Malhotra, G. Marcy, F. Marzari, N. Murray, E. Quataert, and S. Tremaine for encouraging and informative exchanges.

REFERENCES

Armitage, P.J., & Hansen, B.M.S. 1999, Nature, 402, 633

Artymowicz, P. 1998, ASP Conf. Ser. 134, ed. R. Rebolo, E.L. Martin, & M.R. Zapatero-Osorio (San Francisco: Astronomical Society of the Pacific), 152

Bell, K.R., Cassen, P.M., Klahr, H.H., & Henning, Th. 1997, ApJ, 486, 372

Blaes, O.M., & Balbus, S.A. 1994, ApJ, 421, 163

Bryden, G., Rozyczka, M., Lin, D.N.C., & Bodenheimer, P. 2000, ApJ, 540, 1091

Butler, R.P., et al. 1999, ApJ, 526, 916

Chiang, E.I., Tabachnik, S., & Tremaine, S. 2001, AJ, 122, 1607

D'Alessio, P., Calvet, N., Hartmann, L., Lizano, S., & Canto, J. 1999, ApJ, 527, 893

Dermott, S., Malhotra, R., & Murray, C. 1988, Icarus, 76, 295

Fleming, T.P., Stone, J.M., Hawley, J.F. 2000, ApJ, 530, 464

Ford, E.B., Havlickova, M., & Rasio, F.A. 2001, Icarus, 150, 303

Gammie, C.F. 1996, ApJ, 457, 355

Goldreich, P., & Tremaine, S. 1980, ApJ, 241, 425

Goodman, J., & Rafikov, R. 2001, ApJ, 552, 793

Hahn, J.M., & Malhotra, R. 1999, AJ, 117, 3041

Hall, G., & Watt, J.M. 1976, Modern Numerical Methods for Ordinary Differential Equations (Oxford: Clarendon Press)

Henrard, J., & Lemaitre, A. 1983, Celestial Mechanics, 30, 197

Holman, M., Touma, J., & Tremaine, S. 1997, Nature, 386, 254

Kley, W. 2000, MNRAS, 313, 47

Lee, M.H., & Peale, S.J. 2001, ApJ, submitted

Lin, D.N.C., Papaloizou, J.C.B., Terquem, C., Bryden, G., & Ida, S. 2000, in Protostars & Planets IV, ed. V. Mannings, A.P. Boss, & S.S. Russell (Tucson: University of Arizona Press), 1111

Lodders, K., & Fegler, B. 1998, The Planetary Scientist's Companion (New York: Oxford University Press)

Malhotra, R. 1988, Ph.D. thesis, Cornell University, Ithaca, NY

Marcy, G., et al. 2001a, ApJ, 555, 418

Marcy, G., et al. 2001b, ApJ, 556, 296

Murray, C.D., & Dermott, S.F. 1999, Solar System Dynamics (New York: Cambridge University Press)

Murray, N., Hansen, B., Holman, M., & Tremaine, S. 1998, Science, 279, 69

Murray, N., et al. 2001, ApJ, 555, 801

Papaloizou, J.C.B., & Larwood, J.D. 2000, MNRAS, 315, 823

Papaloizou, J.C.B., Nelson, R.P., & Masset, F. 2001, A&A, 366, 263

Peale, S. 1986, in Satellites, ed. J.A. Burns, & M.S. Matthews (Tucson: University of Arizona Press), 159

Rasio, F., & Ford, E.B. 1996, Science, 274, 954

Sinclair, A.T. 1972, MNRAS, 160, 169

Stone, J.M., Gammie, C.F., Balbus, S.A., & Hawley, J.F. 2000, in Protostars & Planets IV, ed. V. Mannings, A.P. Boss, & S.S. Russell (Tucson: University of Arizona Press), 589

Ward, W.R. 1997, ApJ, 482, L211

Weidenschilling, S.J., & Marzari, F. 1996, Nature, 384, 619

Yu, Q., & Tremaine, S. 2001, AJ, 121, 1736

This preprint was prepared with the AAS LaTeX macros v5.0.